MaPS 2024 Junior Syllabus and Schedule

Term 1

Introduction to Proofs

In mathematics, we are always looking at **statements** and interested in whether these statements are **true** or **false**, and why. This introduction emphasises the importance of rigorously proving that a statement is true or false, and what techniques and methods of proof that we can use to achieve this including **Direct Proof**, **Proof by Contradiction** and **Proof by Exhaustion**.

Divisibility

The study of integers, called **Number Theory**, is one of the oldest branches of mathematics and has been of interest to humans dating back to Ancient Mesopotamia, Egypt, Greece, India and so on. One fundamental concept that separates integers from other numbers is the idea of **divisibility**. This seemingly basic idea has far reaching implications that builds the entire field of Number Theory. This topic will introduce the core definitions of divisibility and extend this to the **Fundamental Theorem of Arithmetic**, the **Euclidean Algorithm** and **Bézout's Identity**.

The Pigeonhole Principle

The **Pigeonhole Principle** states that if $n+1$ pigeons are to be distributed among n pigeonholes, then at least one pigeonhole will contain two pigeons. This idea is very obvious at first sight but can be generalised and has surprising applications in many different fields of mathematics including geometry, combinatorics, mathematical analysis and more. In this unit, we will investigate some **applications** of the pigeonhole principle, and extend it to the **infinite case**.

Circles

The **circle** is a fundamental shape that appears everywhere in nature and has been used to inspire the study of geometry, astronomy and calculus. Being such a basic and symmetric shape, circles have many unique properties that have been studied for millennia. This topic will introduce many fascinating results that can proven about circles and their applications to **geometrical problems**.

Term 2

Mathematical Induction

Continuing from the **Introduction to Proofs** topic in term 1, **Mathematical Induction** is another versatile method of proof that can be used to prove results in many contexts including Number Theory, Combinatorics, Geometry and more. We will introduce the idea of Mathematical Induction and show its applicability to a diverse set of problems and extend this to **Strong Induction**.

Modular Arithmetic

As an extension to the **Divisibility** topic covered in term 1, **Modular Arithmetic** is an alternate system of arithmetic that is only concerned with the remainders of numbers after division by a certain modulus. This is a powerful tool in **Number Theory** and can be used to prove many beautiful results. We will go through the definitions and some examples of Modular Arithmetic and investigate results including **Quadratic Residues**, **Fermat's Little Theorem** and the **Chinese Remainder Theorem**.

Inequalities

In the study of algebra, we are often not just interested in the equality of two expressions but also when one expression will always be greater or less than another. This can often be a challenge when we are dealing with unknown pronumerals where will need to use rigorous mathematical techniques to prove that an inequality holds. We will investigate rigorous proofs for inequalities using the fact that **Squares Are Never Negative** and extend this result to important inequalities like the **AM-GM inequality**.

Colourings and Invariants

Many combinatorial problems and games can involve a large (possibly infinite) number of outcomes. Using colourings and invariants are an elegant way to address every outcome to solve these types of problems. We will investigate some different types of colourings and invariants including **Parity Invariants**, **Modular Invariants**, **Colouring Invariants**, **Geometric Invariants**.

Term 3

Methods of Counting

An important task in combinatorics is being able to count the number of ways objects can be arranged or selected. In this unit, we will introduce the core concepts in counting, namely the **Addition Principle** and **Multiplication Principle** and extend these to the idea of **Permutations** and **Combinations** with restrictions.

Polynomials

Polynomials are a special type of mathematical expression with far reaching applications in fields like chemistry, physics, economics and many other areas in mathematics. Their simple definition leads to its versatility in many mathematical problems as well as the development of much theory about them. We will explore the definitions of **polynomials** and extend this to the **Remainder Theorem**, **Factor Theorem** and **Fundamental Theorem of Algebra**.

Lengths and Areas

The interplay between lengths and areas form the backbone of measurement and geometry. It has been studied for millennia and has allowed civilisation to develop to the point it is at today. This topic will introduce fundamental theorems on lengths and areas and discuss their applications. Some theorems that will be covered include **Pythagoras' Theorem**, the **area of a triangle** and **Angle Bisector Theorem**.

Graph Theory

In mathematics, a graph is defined to be a set of points (called **vertices**) which are connected by lines (called **edges**). As you may guess, Graph theory is the study of these objects and given their very broad definition, we are able to prove many interesting results which has many applications in today's society, notably in computer science, biology and social sciences. This unit will introduce the **terminology** to study graphs and some **famous results**.

Term 4

Mathematical Talk Competition

This is an **optional** competition that takes place in late November that will mark the end of the Correspondence Program for the year. You will have the opportunity to work in a group to research a **mathematical topic** and present a 5-10 minute mathematical talk. Further instructions and a list of available topics will be provided at the end of term 3.

You will be given 8 weeks to do the required research, prepare and practice their talk with the guidance of a mentor. **Prizes** may be awarded to the best talk.

Draft Schedule

Term 1

Date	Event
30 January	Term 1 Begins
12 February	Introduction to Proof: notes and problems posted
25 February	Introduction to Proof: problems due
26 February	Divisibility: notes and problems posted
10 March	Divisibility: problems due
11 March	Pigeonhole Principle: notes and problems posted
24 March	Pigeonhole Principle: problems due
25 March	Circles: notes and problems posted
5 April	Term 1 Ends
6 April	Circles: problems due

Term 2

Date	Event
29 April	Term 2 Begins
29 April	Mathematical Induction: notes and problems posted
12 May	Mathematical Induction: problems due
13 May	Modular Arithmetic: notes and problems posted
26 May	Modular Arithmetic: problems due
27 May	Inequalities: notes and problems posted
May	Australian Training Tournament
9 June	Inequalities: problems due
10 June	Colourings and Invariants: notes and problems posted
23 June	Colourings and Invariants: problems due
5 July	Term 2 Ends

Term 3

Date	Event
22 July	Term 3 Begins
22 July	Methods of Counting: Notes and Problems posted
4 August	Methods of Counting: Problems due
5 August	Polynomials: notes and problems posted
18 August	Polynomials: problems due
19 August	Lengths and Areas: notes and problems posted
1 September	Lengths and Areas: problems due
2 September	Graph Theory: notes and problems posted
12 September	AIMO
15 September	Graph Theory: problems due
20 September	Face to face session and Mathematical Talk topics assigned
27 September	Term 3 Ends

Term 4

Date	Event
13 October	Mathematical Talk Outline Due
14 October	Term 4 Begins
27 October	Mathematical Talk First Draft Due
10 November	Mathematical Talk Final Draft Due
November	Mathematical Talk Competition and program close
November/December	Australian Training Tournament
20 December	Term 4 Ends